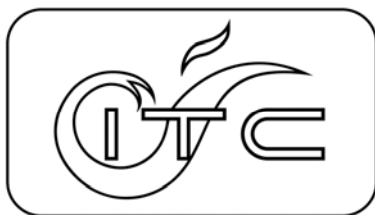


**INTERNATIONAL STUDIES
in SCIENCE and ENGINEERING**

**Numerical Methods
in
Heat Transfer**

Edited by: Andrzej J. Nowak



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International Studies in Science and Engineering

The Editorial Board encourages its colleagues all over the world to publish in the "INTERNATIONAL STUDIES in SCIENCE and ENGINEERING" both text books which accompany a lecture series for students and other books which demonstrate how to apply the knowledge acquired in lecture theatres to industrial practise.

With publishing "Numerical Methods in Heat Transfer" – the eleventh book in this series was released. At least one further book concerning "Advanced Heat Transfer" is expected to be published in 2010.

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Preface

The last couple of decades have seen a substantial increase in the use of mathematical methods for modelling in natural and engineering sciences. Among a great variety of engineering problems that have successfully been coped with, the heat transfer problems belong to the most challenging. They can be considered as separate type of engineering problems offering information how industrial objects should be heated or cooled. However, even more frequently, the heat transfer processes are coupled with other physical processes and this results in so-called multi-physic approach to engineering problems.

As it is widely known the energy consumption in the European Union currently amounts to circa 1500 million tonnes (coal equivalent) of primary energy. By 2020, 1600 million tonnes will be needed even though the energy utilization efficiency is expected to increase by 18%. The changes in the European energy markets as well as the forthcoming Carbon Tax and emission trading schemes are the primary scientific and technological reasons for establishing **INSPIRE network** (optimIzatioN of Systems, energy management, and environmental imPact In pRocess enginEering). The basic network disciplines are economics and process engineering and these are supported by mathematical modelling, including modelling of heat transfer processes, complex environmental systems, fuel processing and combustion.

The aim of publishing this book is to disseminate the knowledge acquired during the execution of the INSPIRE project. Copies of this volume will be distributed among all Early Stage Researchers employed by the network. It should serve as a reference in their further research and engineering career. Copies of the volume have been made available to institutions being the members of the INSPIRE network. The documents should be handy in training of junior staff in numerical techniques. As a matter of fact the numerical methods presented in this book were studied and discussed at numerous workshops of the INSPIRE network. Dr. Gabriel Węcel, who for three years acted as an Experience Researcher of the network responsible for training of the Early Staged Researchers, used these lectures in the CFD training courses.

Contributors to this book are scientists with a competence and a high international reputation in the filed of thermal problems and numerical methods. Five of them are affiliated to the universities forming the INSPIRE Consortium. Prof. Jerzy Banaszek kindly provided chapter four on the finite element method. The complete list of contributors includes:

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To the Student:

How to get the most from these lecture notes

The book does not provide a rigorous and detailed account for the theory of the boundary value thermal problems, so no detailed mathematical proofs are included. Attention is given to practical aspects of the selected numerical methods, and to general techniques rather than specialised procedures. That is why the book is aimed at MSc students, PhD students, researchers and professional engineers, especially to those who are involved in thermal engineering.

It is expected that reader has some theoretical background in the heat transfer processes and is familiar with fundamental mathematics. To study the subject in depth, it is recommended to have also an access to a software capable of solving industrial thermal problems. The most useful would be a number of hands-on-training sessions. A collection of such problems concerning the application of a commercial finite volume package to the solution of CFD conjugate heat transfer problems is available on request form the editor of this book. Please contact Prof. A.J. Nowak, Silesian University of Technology, Institute of Thermal Technology, 44-100 Gliwice, Konarskiego 22, Poland (phone: + 48 32 237 1025, fax: + 48 32 237 2872): andrzej.j.nowak@polsl.pl to get an access to these tutorials.

What's in the book

This book starts from the Chapter 1 where fundamental heat transfer quantities are defined and discussed. Additional to this, governing equations and boundary conditions are also formulated and reviewed.

In the Chapter 2 the most important discretization methods are derived from the so-called weighted residuals. This approach involves point collocation, collocation in subregions, Galerkin formulation and the least squares. Finally, this chapter also explains different weighted functions leading to the finite volume method, the finite element method and the boundary element method.

The next two chapters, *i.e.* Chapter 3 and 4 discuss in detail mathematical background as well as computational aspects of two the most popular numerical techniques know as finite volume method and finite element method. Emphases in these two chapters are on heat transfer problems.

The Chapter 5 is devoted to Computational Fluid Dynamics where coupling of temperature field to at least velocity and pressure fields is made. This chapter covers also the basic information on the most frequently used turbulence models, chemically reacting flows etc.

In the next chapter the mathematical models of thermal radiation are discussed. The problem of coupling radiation with other heat transfer modes is also addressed.

Mathematical modelling of the phase change processes is the subject of the Chapter 7. Classical Stefan condition along with more advanced treatment of the phase change phenomenon are discussed.

The Chapter 8 covers material related to the application of the boundary element method in heat transfer. Analysis starts from the heat conduction problems and then application is extended to heat convection.

The last chapter is devoted to the meshless methods where the solution is obtained without generating any numerical mesh, which means that the solution is obtained at a set of nodal points not connected into elements. The presented version of the meshless method does not require integration over the domain, making this approach very attractive in terms of execution times.

Acknowledgments

I personally feel indebted to all my colleagues – contributors to this book, but also to many anonymous co-workers from the Institute of Thermal Technology, who directly or indirectly contributed to the quality of this work. On behalf of all contributors I would also like to express our sincere appreciation and gratitude to the European Commission for the support received within INSPIRE project financed by the Sixth Framework Programme (MRTN-CT-2005-019296). This support was instrumental in preparing and printing this book.

The cover of the book contains figure provided by Dr. Jacek Smółka of SUT, Gliwice. His offer is gratefully acknowledged.

Although all contributors have made a concerted effort to make this edition error free, some mistakes may have crept in unbidden. I would appreciate hearing from anyone who finds an error or wishes to comment on the text. You may e-mail or write to me.

Gliwice, November 2009

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